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Example 4.1

Find the principal value of $\sin^{-1}(- 1/2)$ (in radians and degrees).

Solution

Let $\sin^{-1}(- 1/2) = y$. Then $\sin y = - 1/2$.

The **range** of the principal value of $\sin^{-1} x$ is $[- \pi/2 , \pi/2]$ and hence, let us find $y \in [- \pi/2 , \pi/2]$ such that $\sin y = - 1/2$. Clearly, $y = - \pi /6$.

Thus, the principal value of $\sin^{-1}(- 1/2)$ is $– \pi/6$. This corresponds to − 30°.

Example 4.2

Find the principal value of $\sin^{-1} (2)$, if it exists.

Solution

Since the domain of $y = \sin^{-1} x$ is [-1, 1] and $2 \in$ [-1, 1] , $\sin^{-1} (2)$ does not exist.

Example 4.3

Find the principal value of

Solution

We know that $\sin^{-1} : [-1, 1] \rightarrow [-\pi/2 , \pi/2]$ is given by

$\sin^{-1} x = y$ if and only if $x = \sin y$ for $-1 \leq x \leq 1$ and $\pi/2 \leq y \leq \pi/2$. Thus,

Example 4.4

Find the domain of $\sin^{-1}(2 - 3x^2)$

Solution

We know that the domain of $\sin^{-1}(x)$ is [-1, 1].

This leads to $-1 \le 2 - 3x_2 \le 1$, which implies $-3 \le -3x_2 \le -1$.

Now, $-3 \le -3x_2$, gives $x_2 \le 1$ and ... (1)

$-3x_2 \le -1$, gives $x_2 \ge 1/3$ ... (2)

Combining the equations (1) and (2), we get $1/3 \le x_2 \le 1$. That is, $1/\sqrt{3} \le |x| \le 1$, which gives

, since $a \le |x| \le b$ implies $x \in [-b, -a] \cup [a, b]$.